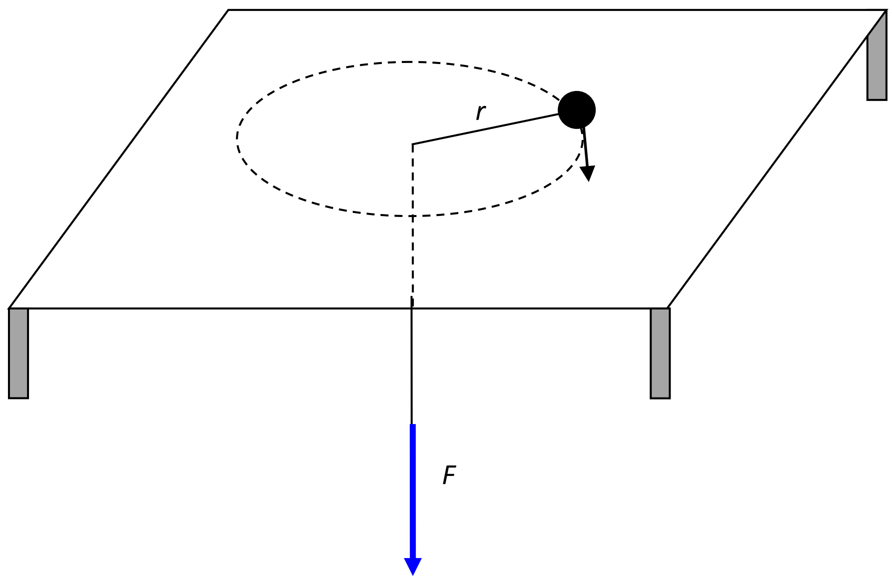
Teacher notes
Topic A

A problem with angular momentum

In the textbook we considered the problem shown in the figure where instead of a force acting on the string, we had a hanging mass. We then asked for the value of that mass to have the hanging mass at rest. Here we have a force acting rather than a hanging mass.

A ball of mass *m* rotates on a circle of radius *r* with kinetic energy *K* on horizontal table. The ball is attached to a string that goes through a hole in the table. A force *F* acts on the string as shown.



(a) Explain why $F = \frac{2K}{r}$.

(b) Show that $K = \frac{L^2}{2mr^2}$, where *L* is the angular momentum of the mass about the vertical axis.

(c) Find an expression for the force in terms of *L*, the angular momentum of the mass about the vertical axis.

(d) State the torque of *F* about an axis going through the vertical string.

(e) Explain why $L = \text{constant}$.

(f) The force is increased so that the radius of the circular path halved. Calculate the new force.

(g) Calculate the work done by the force in halving the radius of the circle using

(i) the work kinetic energy relation,

(ii) the definition of work (you must use calculus).

Answers

(a) $F = \text{tension} = \frac{mv^2}{r} = \frac{2K}{r}$.

(b) $K = \frac{1}{2}mv^2 = \frac{1}{2}m(\frac{L}{mr})^2 = \frac{L^2}{2mr^2}$.

(c) $F = \frac{2K}{r} = \frac{1}{r} \times \frac{2L^2}{2mr^2} = \frac{L^2}{mr^3}$.

(d) It is zero.

(e) The net torque on the system is zero.

(f) $F' = \frac{L^2}{mr'^3} = \frac{L^2}{m(\frac{r}{2})^3} = 8F$.

(g)

(i) $W = \Delta E_K = \frac{1}{2}mu^2 - K$. Since $L = \text{constant}$ we know that $mvr = mu\frac{r}{2} \Rightarrow u = 2v$. Hence

$W = 4K - K = 3K$.

(ii) Let l be the length of the string. Then the hanging length is $x = l - r$ so that $r = l - x$. Then,

$$W = \int_{l-r}^{l-\frac{r}{2}} F dx = \int_{l-r}^{l-\frac{r}{2}} \frac{L^2}{m(l-x)^3} dx = + \frac{L^2}{2m(l-x)^2}\Bigg|_{l-r}^{l-\frac{r}{2}} = \frac{4L^2}{2mr^2} - \frac{L^2}{2mr^2} = \frac{3L^2}{2mr^2} = 3K.$$